

Querying Bayesian model output with PostgreSQL

Wellington PostgreSQL Users Group

Finlay Thompson

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Bayesian modelling



Bayesian analysis involves converting data and conceptual models into probability distributions.

We interpret probabilities in the broad sense that:

- a probability p is a number between 0 and 1
- where $p = 1$ corresponds to TRUE
- and $p = 0$ corresponds to FALSE
- probabilities measure our confidence in a statement

Note that traditional 20th century statistics understands probabilities as the ratios coming from repeated experiments (think coin tosses).

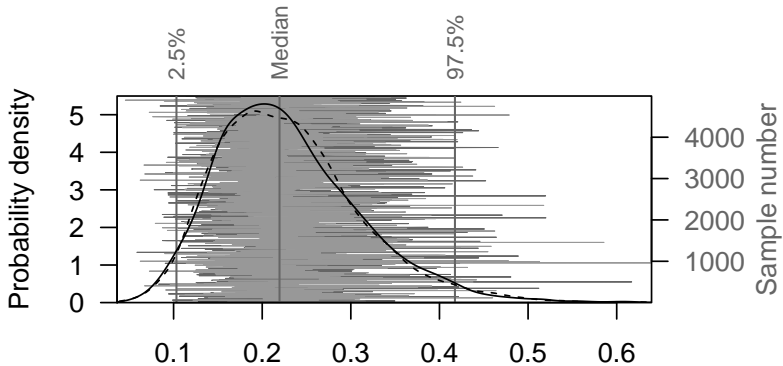
D represents the data, and is typically records in a database.

θ represents the parameters of some kind of model.

$$P(\theta|D) = \frac{P(\theta)P(D|\theta)}{P(D)}$$

$P(D|\theta)$ is the *likelihood* distribution.
Probability of measuring D given θ .

$P(\theta|D)$ is the *posterior* distribution.
It represents the result of fitting the data D to the model θ .

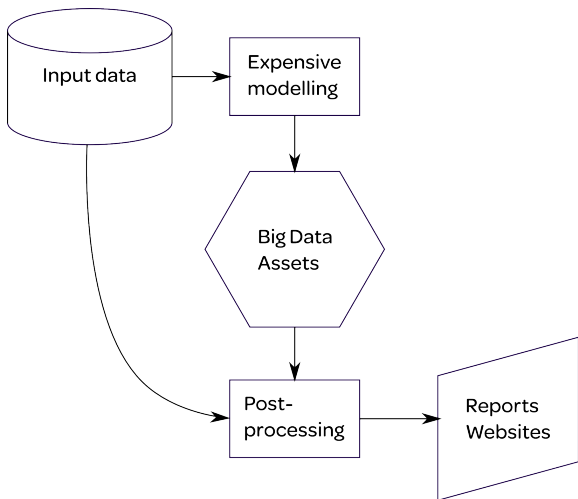


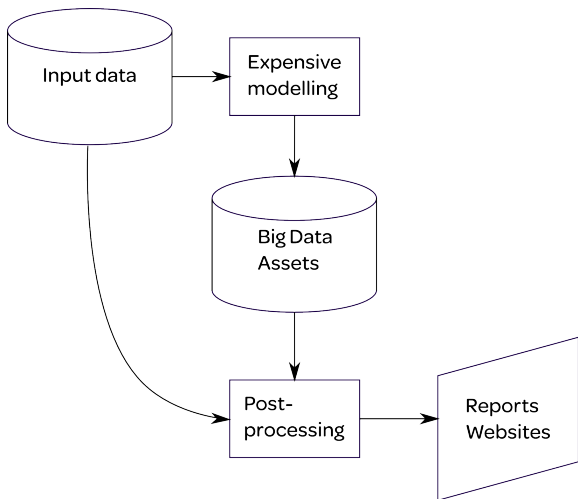
We use Monte Carlo Markov chain (MCMC) methods that are slow, and produce lots of output.

MCMC methods are **accurate**.

The posterior output is in the form of *samples from the posterior*. In practice this means 4000 values per parameter.

Typically models have hundreds of parameters.



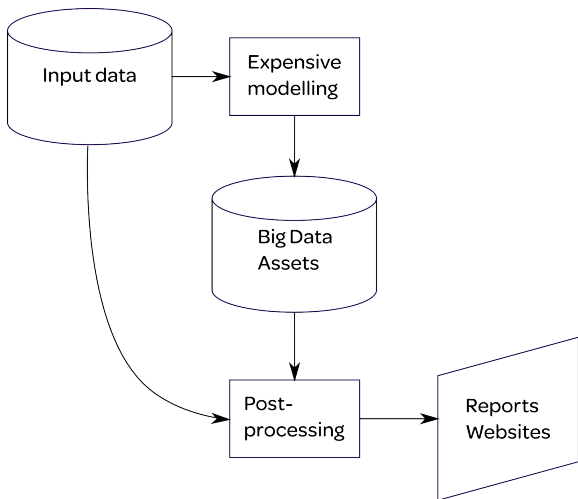


Another advantage of MCMC methods is **flexibility**.

We use the parameter samples to calculate samples for every desired output.

For example, we can produce 4000 samples for each of approx 1.5 million fishing events.

This output is big and expensive. So we moved it into PostgreSQL.



Storing and querying distributions

The estimates are stored in the form of arrays of integers.

They represent *uncertain* quantities, with each value in the array a realisation from the (unknown) posterior distribution.

The order of the arrays are significant, preserving the correlation structure of the estimates.

```
CREATE TABLE estimate (  
    model_id    INTEGER REFERENCES model(id),  
    effort_id   INTEGER NOT NULL,  
    observed    INTEGER, -- null if effort not observed  
    estimate    INTEGER[] NOT NULL  
);
```

This results in storing a large quantity of data.

Currently around 12 GB

Need to aggregate the data, so integer array sums!

The standard function for aggregating integer arrays is a bit slow.

It checks the lengths of the arrays, and checks the types of arguments, checks checks checks.

I took the library function and ripped the checking out!

Open source for the win!


```
#include "postgres.h"
#include "fmgr.h"
#include "utils/array.h"

#ifdef PG_MODULE_MAGIC
PG_MODULE_MAGIC;
#endif

Datum int_array_sum(PG_FUNCTION_ARGS);

PG_FUNCTION_INFO_V1(int_array_sum);
Datum
int_array_sum(PG_FUNCTION_ARGS) {

    ArrayType * state = PG_GETARG_ARRAYTYPE_P(0);
    ArrayType * new = PG_GETARG_ARRAYTYPE_P(1);

    int numargs = ARR_DIMS(state)[0];
    int * state_ptr = (int *) ARR_DATA_PTR(state);
    int * new_ptr = (int *) ARR_DATA_PTR(new);

    int i;
    for (i = 0; i < numargs; i++)
        state_ptr[i] += new_ptr[i];

    PG_RETURN_ARRAYTYPE_P(state);
}
```

```
SET search_path = public;

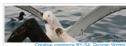
CREATE OR REPLACE FUNCTION int_array_sum(int[], int[])
RETURNS int[]
AS '$libdir/intarraysum', 'int_array_sum'
LANGUAGE C IMMUTABLE STRICT;

DROP AGGREGATE IF EXISTS sum(int[]);
CREATE AGGREGATE sum (int[]) (
    SFUNC = int_array_sum,
    STYPE = int[]
);
```

Result is queries that are in the order of 1000 times faster.

The result is much more flexibility in reporting, and happy clients.

Capture of all birds in trawl fisheries



Leach's booby (L. E.A. Deane & Stanger)

Observed captures of all birds in trawl fisheries



Fishing effort and observations in trawl fisheries



Estimated captures of all birds in trawl fisheries

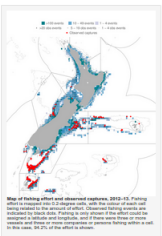


Fishing effort and observed captures of all birds by month



New Zealand is a centre of seabird diversity, with over 80 species breeding in the New Zealand region. Seabirds are frequently targeted on fisheries or fisheries with vessel-regulated capture being either of abundance (birds), abundance or weight (fishery management). Current monitoring is based on commercial fisheries.

In the 2012-13 fishing year, there were 100 observed captures of all birds in trawl fisheries. Observed captures were of white-rumped petrels (20), New Zealand shearwaters (20), Sooty shearwaters (17), muttonbirds (16), Buller's shearwaters (17), Sooty's shearwaters (17), Sooty petrels, petrels and shearwaters (21), petrels, petrels, and shearwaters (10), grey petrels (5), shearwaters (2), albatrosses (2), great shearwaters (4), very petrels (4), muttonbirds (10), petrels, petrels, and shearwaters (10), shearwaters (5), shearwaters (2), Phoebastria petrels (2), Cape petrels (2), petrels (1), muttonbird (1), and shearwaters (1). It was estimated by a statistical model that there were a total of 2,624 (95% c.i. 2,025-3,405) captures in trawl vessels.



Map of fishing effort and observed captures, 2002-12. Fishing effort is measured each 15-minute cycle, with the colour (black and blue) representing the amount of time that vessels fished (black) and blue) and the size of the dots representing the number of birds captured (black dots). Fishing is only shown if the effort could be assigned a latitude and longitude, and if there were three or more vessels and three or more captures or persons fishing within a cell. In this case, 88.2% of the effort is shown.

Table of effort, captures, and estimated captures by fishing year. For each fishing year, the table gives the total number of tows, the total of observed captures, the total number of tows, the estimated number of birds, the mean number of birds per tow, the mean number of birds per tow, and the percentage of tows included in the estimate. Due to the way the tows are sampled, the number of tows included in the estimate is only shown if there were three or more vessels and three or more captures or persons fishing in that year. For more information on the methods used to generate the table, see Johnston and Thompson (2012).

Fishing effort	Observed captures		Estimated captures					
	All tows	Observed tows	Number	Rate	Mean	95% c.i.	% tows included	
2002-03	130 181	8 828	5.3	299	3.63	2 311	2 543-4 149	100.0
2003-04	122 844	6 247	5.1	362	4.26	2 763	2 120-3 684	100.0
2004-05	120 439	7 712	6.4	483	6.28	4 539	3 486-6 080	100.0
2005-06	169 945	6 619	6.0	399	5.38	3 595	2 773-4 630	100.0
2006-07	162 287	7 820	5.7	213	2.85	2 249	1 774-2 925	100.0
2007-08	89 524	6 945	10.1	234	2.58	1 895	1 470-2 385	100.0
2008-09	87 945	6 804	11.2	489	4.78	2 480	2 020-3 020	100.0
2009-10	82 588	6 008	8.7	269	2.88	2 023	1 582-2 674	100.0
2010-11	86 580	7 443	8.6	362	4.88	2 468	1 980-3 121	100.0
2011-12	84 439	6 955	10.8	248	2.73	1 863	1 440-2 387	100.0
2012-13	83 723	12 383	14.8	359	5.72	2 604	2 040-3 465	100.0

Download CSV

Table of observed captures, during the 2012-13 fishing year. The table lists the protected species captures that were recorded by observers. For each capture, the table gives the date and time, the best identification made (species), the status of the animal (Alive, Dead, Wound, or Discarded), the identification method (Photograph, Photograph, Inspection, Observer), the capture method, the species targeted by the fishing, and the area where the capture occurred.

Date	Species or species group	Status	Identification	Capture method	Target species	Area
09 Oct 2012 23:09	Sooty's shearwater	Dead	Necropsy	Net	(None)	East Coast South Island
14 Oct 2012 08:55	Sooty shearwater	Dead	Necropsy	Net	(None)	East Coast South Island

Reporting is possible in many different slices:

- Fishing year
- target species of fishers
- reporting areas
- type of vessel

Each of these estimates has an accurate estimate of uncertainty, with published confidence intervals.

For a publicly visible example, see <https://data.dragonfly.co.nz/psc/>.